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- USSR -

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A SIMPLE FLOATING PENDULUM GYROCOMPASS WITH SUSPENSION ON A CORE BEARING

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At the present time several experimental models have been made of land gyroscopic compasses, whose sensing components are suspended in a liquid and centered on a core bearing (Fig. 1). We cite below a theoretical investigation of the motion of the sensing component of this compass at an arbitrary position of the center of gravity and of the center of volume.

In order to compose differential equations of motion, we will use a geographically-oriented system of coordinates of $O\xi\eta\zeta$. We will direct axis ξ along the meridian line northward, axis ζ toward the zenith, and the direction of axis η is determined by the selection of the right-hand system of coordinates (Fig. 2).

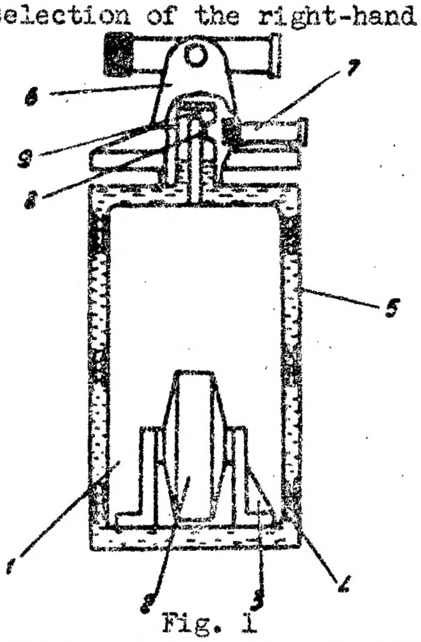

Fig. 1

Structural diagram of floating gyrocompass with suspension on a core bearing: 1-sensing component; 2-gyromotor; 3-attachment brackets of gyromotor; 4-electrodes; 5-frame; 6-theodolite; 7-collimator tube; 8-mirror; 9-core bearing.

The vertical and horizontal components of the diurnal rotation of the earth about axis ζ and ξ will be correspondingly U sin φ, $U\cos\varphi$ where φ is the latitude of the location.

We will connect the system of coordinates OXYZ with the sensing component. We will make point O coincide with the point of suspension. Axis Y in the position of equilibrium of the sensing component and when the gyroscope does not rotate (H = 0) coincides with the vertical, and then the remaining axes X and Z lie in the plane of the horizon.

Let us assume that the principal axis of the gyroscope is directed along with or parallel to axis Z. The coordinates of the center of gravity of the sensing component in the system of coordinates OXYZ will be l_x, l_y, l_z, and the coordinates of the center of volume will be L_x, L_y, L_z.

In a position of equilibrium of the sensing component and at H = 0, the following conditions are present:

$$
\begin{aligned}
M_x &= Pl_z - QL_z = 0. \\
M_z &= QL_x - Pl_x = 0. \\
\Sigma P_y &= Q - P - R = 0.
\end{aligned} \qquad (1)
$$

where P is the weight of the sensing component;
Q is its carrying capacity;
R is the reaction of the core bearing; and
M_x, M_y are external torques acting relative to axes X and Y.

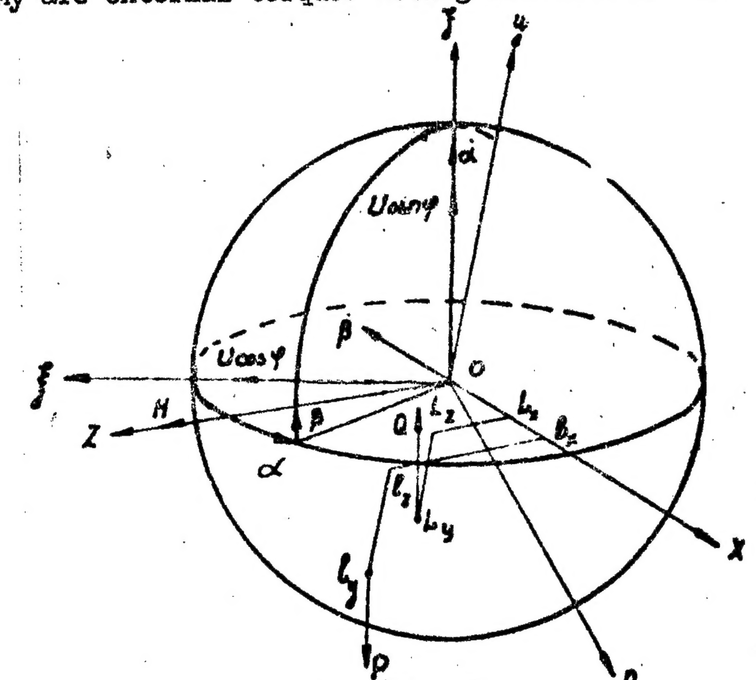

Fig. 2

Angles determining the position of the sensing component.

Let us assume that the instantaneous position of axis Z (at $H \neq 0$) relative to the system of coordinates $O\xi\eta\zeta$ are determined by angles α and β and by angular velocities $\dot{\alpha}$ and $\dot{\beta}$.

The sum total of projections of external and gyroscopic torque on the axes Y and X will be correspondingly:

$$\left.\begin{aligned} \kappa\dot{\alpha} + HU\cos\varphi\cdot\alpha + H\dot{\beta} + (QL_x - Pl_x)\beta = 0 \\ \kappa\dot{\beta} + (Pl_y - QL_y)\beta - H\dot{\alpha} = HU\sin\varphi + QL_z - Pl_z \end{aligned}\right\}, \tag{2}$$

where k is the coefficient of friction with the liquid (the forces of friction in the core bearing are not taken into account).

If we neglect moment $k\dot{\beta}$, as being very small, then on condition (1) we will rewrite Equations (2) as follows:

$$\left.\begin{aligned} \kappa\dot{\alpha} + HU\cos\varphi\cdot\alpha + H\dot{\beta} = 0 \\ (Pl_y - QL_y)\beta - H\dot{\alpha} = HU\sin\varphi \end{aligned}\right\} \tag{3}$$

The position of equilibrium of axis z of the sensing component is determined by the expressions

$$\alpha_r = 0,\ \beta_r = \frac{HU\sin\varphi}{Pl_y - QL_y}.$$

Excluding the variable β from the first equation, we get

$$\ddot{\alpha} + 2h\dot{\alpha} + \mu^2\alpha = 0, \tag{4}$$

where

$$2h = \frac{\kappa(Pl_y - QL_y)}{H^2},\quad \mu^2 = \frac{(Pl_y - QL_y)\,U\cos\varphi}{H}. \tag{5}$$

The total integral of Equation (4) is

$$\alpha = De^{-ht}\cos\left(\sqrt{\mu^2 - h^2}\,t - \psi\right).$$

The law of motion of the sensing component will be determined at the following initial conditions: let at $t = 0$, $\alpha = \alpha_c$, $\dot{\alpha} = 0$. As a result of calculations made, we will obtain

$$\alpha = \alpha_c\frac{\mu}{n}e^{-ht}\cos(nt - \psi) \approx \alpha_c e^{-ht}\cos(nt - \psi), \tag{6}$$

where

$$n = \sqrt{\mu^2 - h^2},\quad \operatorname{tg}\psi = \frac{h}{n}. \tag{7}$$

In the land floating gyrocompasses with suspension on a core bearing the weight of the sensing component is practically equal to the carrying capacity, i.e. $P \approx Q$. Therefore the period of free undamped oscillations can be determined with sufficient accuracy according to the formula

$$T = 2\pi\sqrt{\frac{H}{P(l_y - L_y)\,U\cos\varphi}}. \tag{8}$$

It follows from expression (8) that the magnitude of the period of free undamped oscillations at $p \approx Q$ depends, in particular, on the distance between the center of gravity of the sensing component and the center of its volume. In order to diminish the magnitude of the period of free undamped oscillations and hence to reduce the time, which is necessary to determine the gyroscopic azimuth of the orienting side, in designing the apparatus the center of the volume of the sensing component should, if possible, be removed from its center of gravity.

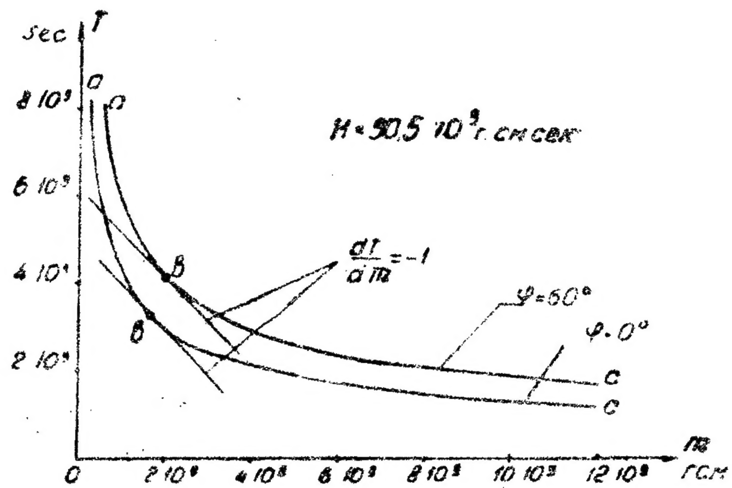

Fig. 3
Dependence of the magnitude of the period of free undamped oscillations of the sensing component on the erection torque.

Consequently, with an identical weight of the sensing component, the magnitude of the period will depend on the nature of the distribution of its volume relative to the center of gravity, but it is not affected by the position of the point of suspension.

However, a diminishing of the period of free undamped oscillations of a simple pendulum gyrocompass due to an increase of the erection torque $\mathfrak{M}$ is indicated within a determined range. This is directly evident from the curve $T = f(\mathfrak{M})$ (Fig. 3) where in a particular instance the magnitude of $\mathfrak{M}$ can be

$$\mathfrak{M} = P\,(l_y - L_y).$$

The cited functions indicate that a diminishing of the magnitude of the period with an increase of the erection torque initially proceeds quite intensively. This takes place on condition $\frac{dT}{d\mathfrak{M}} < -1$. curve sector ab. Subsequently, with an

increase of the erection torque into equal magnitudes the diminishing of the period takes place at an actually smaller magnitude, i.e., for a small diminishing of the period it is necessary considerably to increase the size of the apparatus, the sector of curve bc, and then $\frac{dT}{d\mathfrak{M}} > -1$.

It is therefore indicated to have such parameters of the apparatus which would satisfy the relation $\frac{dT}{d\mathfrak{M}} \leqslant -1$, or, substituting the values of period (8), we find

$$\mathfrak{M} \leqslant 51.3 \sqrt[3]{\frac{H}{\cos\varphi}} \approx 50 \sqrt[3]{\frac{H}{\cos\varphi}}. \qquad (9)$$

where H and have, correspondingly the dimensions Γcm sec, Γ cm.

The above dependence (9) between the magnitude of the erection torque and the gyroscope's kinetic moment makes it possible to determine fairly rapidly whether the parameters of the apparatus correspond to the sector ab of curve $T = f(\mathfrak{M})$ or whether they reach beyond it.

We will analyze the motion of the sensing component at a variable magnitude of its carrying capacity. Let us assume that the magnitude of the carrying capacity was initially constant $Q = Q_0$ and at the same time condition (1) existed. Subsequently, as a result of the change of density of the supporting liquid, caused in principle by the instability of the temperature regime, the carrying capacity will, within a small time interval $t = t_1$ change according to the arbitrary law $Q = Q_0 - q(t)$, while $t_1 \ll T$ and $q(t) \ll Q_0$.

Substituting the value of Q into the system of equations (2) and taking into account equalities (1) we will obtain the equations of motion of the sensing component, whose carrying capacity changes according to the arbitrary law

$$\left.\begin{aligned} &\kappa\dot{\alpha} + HU\cos\varphi\cdot\alpha + H\dot{\beta} - L_x q(t)\cdot\beta = 0 \\ &\kappa\dot{\beta} + (Pl_y - Q_0 L_y)\beta + q(t)L_y\beta - H\dot{\alpha} = HU\sin\varphi - L_z q(t) \end{aligned}\right\} \qquad (10)$$

Determining the value of β from the second equation and substituting it into the first equation (we neglect moment kβ as being very sm ll) we have

$$\ddot{\alpha} + \left[2h + 2h\nu q(t) - \frac{L_x}{H} q(t) - \frac{L_y \dot{q}(t)}{\mathfrak{M} + L_y q(t)}\right]\dot{\alpha} + [\mu^2 + \mu^2\nu L_y q(t)]\alpha =$$

$$= \frac{L_y U \sin\varphi\, \dot{q}(t)}{\mathfrak{M} + q(t)} + \frac{L_z}{H}\dot{q}(t) + \frac{L_x U \sin\varphi\, q(t)}{H} \quad \frac{q(t)\,\dot{q}(t) L_z}{H[\mathfrak{M} + q(t)]} - \frac{L_x L_y q(t)\,\dot{q}(t)}{H^2} \qquad (11)$$

where $\mathfrak{M} = Pl_y - QL_y$ is the erection torque at $Q = Q_0$;

$\nu = \frac{1}{\mathfrak{M}}$ is a small parameter;

$\mu^2 = \frac{\mathfrak{M} U \cos\varphi}{H}$ is the frequency of free undamped oscillations at $Q = Q_0$;

and $2h = \frac{\varkappa \mathfrak{M}}{H^2}$ is the doubled damping coefficient at $Q = Q_0$.

Expanding the terms of Equation (11) into a series containing in the denominator $\mathfrak{M} + q(t)$ according to the exponents and neglecting the terms of the second and higher degree of smallness, we will obtain

$$\ddot{\alpha} + [2h + a\nu q(t) - L_y \nu \dot{q}(t)]\,\dot{\alpha} + [\mu^2 + \mu^2 \nu L_y q(t)]\,\alpha = b q(t) + c q(t), \tag{12}$$

where

$$2h - \frac{\mathfrak{M}}{H} L_x = a, \quad \frac{\beta_1 L_y + L_x}{H} = b,$$

$$\beta_1 = \frac{HU \sin\varphi}{\mathfrak{M}}, \quad c = \frac{L_x U \sin\varphi}{H}. \tag{13}$$

We solve Equation (12) by the method of successive approximations, and for this purpose we will transfer the terms which contain variable coefficients with unknowns, into the right-hand side of the equality, namely

$$\ddot{\alpha} + 2h\dot{\alpha} + \mu^2\alpha = bq(t) + cq(t) + \nu\{[L_y q(t) \quad a q(t)]\,\alpha + \mu^2 L_y q(t)\,\alpha\} \tag{14}$$

We will take as the equation of zero approximation

$$\ddot{\alpha}_0 + 2h\dot{\alpha}_0 + \mu^2\alpha_0 = 0. \tag{15}$$

Substituting the solution of Equation (15) into (14), we will obtain the equation of the first approximation

$$\ddot{\alpha}_1 + 2h\dot{\alpha}_1 + \mu^2\alpha_1 = bq(t) + cq(t) + \nu\{[L_y q(t) - a q(t)]\,\dot{\alpha}_0 - \mu^2 L_y q(t)\,\alpha_0\} \tag{16}$$

Similarly, the equation of the n-th approximation will be

$$\ddot{\alpha}_n + 2h\dot{\alpha}_n + \mu^2\alpha_n = bq(t) + cq(t) + \nu\{[L_y q(t) - a q(t)]\,\dot{\alpha}_{n-1} \quad \mu^2 L_y q(t)\,\alpha_{n-1}\}$$

Then we will find the solution of Equation (12) as the sum total

$$\alpha = \alpha_0 + (\alpha_1 - \alpha_0) + (\alpha_2 - \alpha_1) + \cdots,$$

whence

$$\alpha = \lim_{n\to\infty} \alpha_n$$

In our investigation we limit ourselves only to the solution of the equation of the first approximation, i.e., we assume

$$\alpha \approx \alpha_1. \qquad (17)$$

The particular solution of the equation of zero approximation, at initial conditions t = 0, $\alpha = \alpha$, $\alpha = 0$ will be

$$\alpha_0 = \alpha_0 \frac{\mu}{n} e^{-ht} \cos(nt - \psi) \approx \alpha_0 e^{-ht} \cos(nt - \psi). \qquad (18)$$

where n and have values determinable by expressions (7).

Substituting solution (18) into Equation (16), we will obtain

$$\ddot{\alpha}_1 + 2h\dot{\alpha}_1 + \mu^2 \alpha_1 = N(t), \qquad (19)$$

where

$$N(t) = b\dot{q}(t) + cq(t) + \alpha_c \nu \mu e^{-ht} \{[aq(t) - L_y \dot{q}(t)] \sin nt - \mu L_y q(t) \cos(nt - \psi)\}.$$

The total integral of Equation (19) will be

$$\alpha_1 = e^{-ht}(c_1 \cos nt + c_2 \sin nt) + \frac{1}{n} \int_0^t e^{-h(t-\tau)} N(\tau) \sin n(t-\tau) d\tau. \qquad (20)$$

It should be noted that in order to prevent uncertainty, the letter stands under the sign of the integral instead of t.

We will determine the particular solution of Equation (19) at the accepted initial equations: at t = α_1, $\alpha_1 = \alpha_c$, $\alpha_1 = 0$. As a result of the calculations made the arbitrary constants have the following values:

$$c_1 = \alpha_c, \quad c_2 = \alpha_c \frac{h}{n}.$$

Substituting the values of arbitrary constants into Equations (20) and taking into account the relation (17), we will obtain the solution of Equation (12) with an accuracy up to the terms which contain the magnitude $\nu q(t)$, in the first degree

$$\alpha = \alpha_c e^{-ht} \cos(nt - \psi) + \frac{b}{n} \int_0^t e^{-h(t-\tau)} \dot{q}(t) \sin n(t-\tau) d\tau +$$

$$+ \frac{c}{n} \int_0^t e^{-h(t-\tau)} q(t) \sin n(t-\tau) d\tau + \alpha_c \nu \frac{\mu}{n} e^{-ht} \int_0^t N_1(\tau) \sin n(t-\tau) d\tau, \qquad (21)$$

where $N_1(\tau) = [aq(\tau) - L_v \dot{q}(\tau)] \sin n\tau - \mu L_v q(\tau) \cos(n\tau - \psi)$.

The position of equilibrium of land gyroscopic compasses is determined according to the extreme displacements of the sensing component in the azimuth, but which are not binding due to unavoidable chance disturbances. As a result, an error appears in the reading corresponding to the position of equilibrium. As we shall point out below, the magnitude of the error

depends on the nature of chance disturbances. In this article we are concerned only with the qualitative aspect of this phenomenon.

From the obtained approximate law of motion of the sensing component (21), it follows that during arbitrary changes of its carrying capacity chance disturbances are superposed on the free damped oscillations $a_c e^{-ht}\cos(nt - \psi)$ which are caused by the instability of the carrying capacity, and are determined by the consequent terms. The disturbed motion distorts the regularity of oscillation of the sensing component in the azimuth, and there appears an error in reading off its position of equilibrium.

If the carrying capacity of the sensing component is constant, i.e., if the temperature of the supporting liquid is unchanging, then $Q = Q_0$ and $q(t) = 0$. There is no disturbed motion during this condition. The succession of extreme displacements of the sensing component in the azimuth will be regular and it is determined in accordance with expression (6). Here the nature of oscillations of the sensing component, not containing any chance disturbances, does not introduce errors into the reading off of the position of equilibrium.

In order to obtain a sufficiently accurate reading from the apparatus, it is necessary to diminish the magnitude of disturbed motion caused by the instability of the carrying capacity of the sensing component. However, a high degree of temperature stability of the supporting liquid is not enough to ensure minimum values of the magnitude of disturbed motion. As follows from expression (21), it is also necessary to make sure that the cofactors b, c, and a are sufficiently small. In accordance with designation (13) the values of b, c, and a depend on the design constants of the apparatus, and in particular on the coordinates of the center of volume of the sensing component (L_x, L_y, L_z) within the system of coordinates OXYZ. Therefore, in order to diminish the disturbed motion it is necessary to make sure that the center of volume of the sensing component should sufficiently accurately coincide with the point of its suspension. It should be noted that in the ideal case, at $L_x = L_y = L_z = 0$ there is no disturbed motion caused by the instability of the carrying capacity and the oscillations of the sensing component are described by the expression (6).

At the present time the sensing components of such gyrocompasses are balanced relative to the rotational axis Y. In balancing, attempts are made to distribute the center of gravity and the center of volume along axis Y, i.e., to have $L_x = 0$ and $L_z = 0$. However, such balancing ensures only a partial diminution of disturbed motion caused by the instability of the carrying capacity. As a result of the fact that the center of volume of the sensing component does not coincide with the

point of suspension, i.e. $L_y \neq 0$, the magnitude of cofactor b can be considerable. At the same time the disturbed motion will produce a substantial distortion of the regular oscillations of the sensing component in the azimuth.

The last components which describe the disturbed motion in expression (21) are directly proportional to the magnitude of amplitude α_c. This component is very small in comparison with terms which contain cofactors b and c. Therefore, disturbed motion which depends on the magnitude of the amplitude will not substantially affect the regular oscillations of the sensing component. Still, it should be borne in mind that it is preferable to determine the position of equilibrium according to small amplitudes than according to large amplitudes.

Conclusions

The investigation of the motion of the sensing component of a floating simple pendulum gyrocompass with suspension on a core bearing at an arbitrary position of the center of gravity an and of the center of its volume, makes it possible to arrive at the following conclusions:

1. The period of free undamped oscillations at is determined according to the formula

$$T = 2\pi \sqrt{\frac{H}{P(l_y - L_y)\cos\varphi}}.$$

The magnitude of the period depends on the distance between the center of gravity of the sensing component and the center of its volume and is not affected by the position of the point of suspension.

2. In order to diminish disturbed motion caused by the instability of the carrying capacity, and hence in order to increase the accuracy of reading of the apparatus, it is necessary to make sure that the center of volume of the sensing component should coincide with the point of suspension. The position of equilibrium of the sensing component is preferably determined by small amplitudes than by large ones.

Recommended by the Chair of Gyroscopics

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